Matrix fracture strength in bonded brittle composites

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It has been recognized that the fiber/matrix interface plays a key role on the mechanical performance of fiber reinforced brittle matrix composites [1, 2] (e.g., matrix cracking strength, debonding strength and work of fracture etc.). On the problem of matrix cracking modeling for bonded fibers, Budiansky, Hutchinson, Evans (BHE) [3] investigated the matrix cracking stress caused by the crack-tip transverse tensile stress (see Fig. 1). The BHE analysis considers that the debonded region caused by the crack-tip transverse tensile stress is separated (i.e. τ_s=0) and assumes the debonded length is unchanged as the matrix crack continually propagating. The possible interfacial debonding process in the crack-wake region was not considered in their modeling. By treating the crack-wake debonding as a particular crack propagating problem, Chiang [4] extended Aveston, Cooper and Kelly (ACK) [5] model to include the effect of crack-wake debonding on the stress for onset of steady-state matrix cracking. However, as similar to the ACK model that neglects the elastic response above the slipping region, the Chiang result cannot approach to the no-debond result by Aveston and Kelly (AK) [6] as the no-debond condition is met. In this paper, the BHE model that takes into account the elastic response above the slipping region is extended to include the effect of crack-wake debonding for bonded brittle composites. A fracture mechanics approach in which the crack-wake debonding is treated as a particular crack propagation problem is adopted in the present analysis. Similar to the BHE model that provides results to bridge the extensive slip result by ACK and the no-slip result by AK, the present analysis provides the link between the result of frictionless bonded interface by Stang and Shah [7] and the no-debond result by AK. The difference between the present analysis and Sutcu and Hillig [8], in which the debonding mechanics is approached by energy balance method, will be discussed regarding mathematical modeling and theoretical results.

The composite with fiber volume fraction V_f loaded by a remote uniform stress σ normal to a semi-infinite crack plane is shown in Fig. 1. The downstream region (see Fig. 1) is sufficiently behind the crack-tip so that the stress and strain fields are uniform with respect to the crack plane. In the debonded length (i.e. $0 \leq z l_d$), the fiber-matrix interface is resisted by a constant frictional shear stress τ_s and the fiber and matrix axial stresses are governed by

$$\sigma_f^D(z) = \frac{\sigma}{V_f} - \frac{2\tau_s}{a} z \tag{1}$$

$$\sigma_m^D(z) = \left(\frac{V_f}{V_m}\right) \frac{2\tau_s}{a} z \tag{2}$$

where a is fiber radius and V_m (=1-V_f) is the matrix volume fraction.

A fiber/matrix debonding process in the downstream region is schematically shown in Fig. 2, in which a debonded fiber is loaded by tractions T and τ_s with corresponding fiber and matrix displacements dv and dw, respectively. Following the arguments of Gao, Mai and Cotterell [9] and Stang and Shah that the fracture mechanics approach is preferred to the shear stress approach for the interfacial debonding problem, the fracture mechanics approach is adopted in the present analysis. The stresses in the boned length (i.e. $z \geq l_d$) were given by Chiang [10]:

$$\sigma_f^D(z) = \frac{2K\tau_s}{\rho} e^{-\rho(z-l_d)/a} + \frac{E_f}{E}\sigma + \sigma_f^I \tag{3}$$

$$\sigma_m^D(z) = -\frac{2K\tau_s}{\rho}\left(\frac{V_f}{V_m}\right) e^{-\rho(z-l_d)/a} + \frac{E_m}{E}\sigma + \sigma_m^I \tag{4}$$

$$\tau_i^D(z) = K\tau_s e^{-\rho(z-l_d)/a} \tag{5}$$

where σ_f^I and σ_m^I are, respectively, the residual axial stresses in the fiber and the matrix, which satisfy

$$V_f\sigma_f^I + V_m\sigma_m^I = 0 \tag{6}$$

The remaining quantities E_f and E_m are the fiber and matrix Young's moduli and E is the effective axial Young's modulus of composite which can be approximated by the rule-of-mixtures

$$E = V_f E_f + V_m E_m \tag{7}$$

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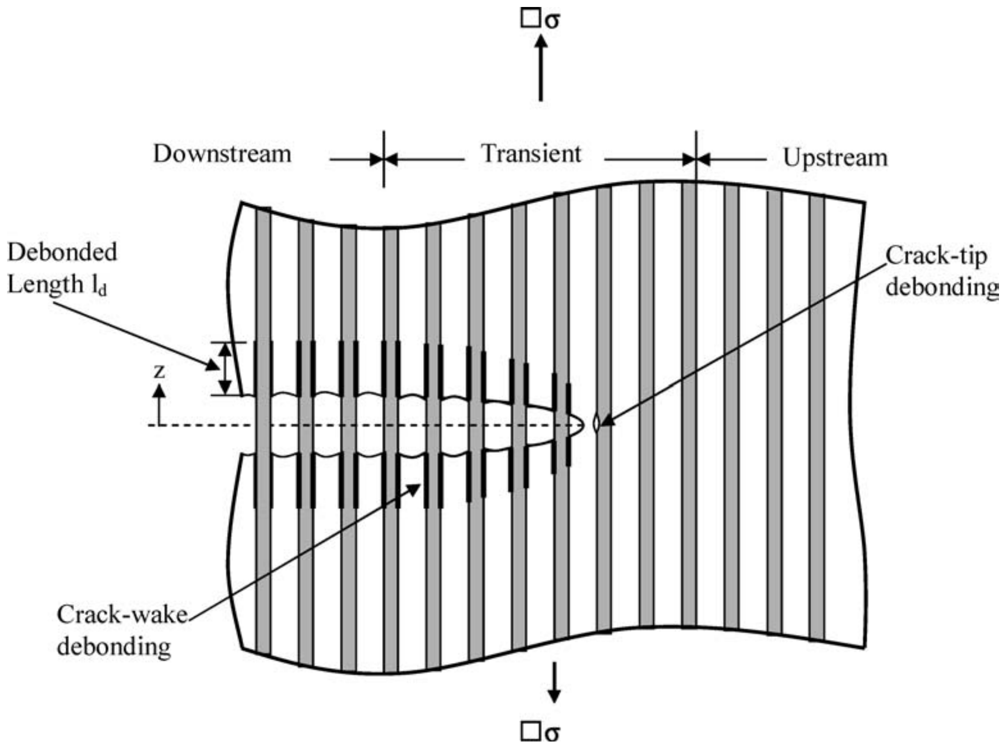

Figure 1 Schematic representation of crack-tip and crack-wake debonding.

The debonded length l_d in Equations (3–5) is given by

$$l_{\rm d} = \frac{aV_{\rm m}E_{\rm m}}{2V_{\rm f}E\tau_{\rm s}}\left(\sigma + (E/E_{\rm m})\sigma_{\rm m}^{\rm I}\right) - \frac{a}{\rho}K \tag{8}$$

where

$$\rho = \sqrt{\frac{2G_{\rm m}E}{V_{\rm m}E_{\rm m}E_{\rm f}\beta}} \tag{9}$$

The quantity of β, which depends only on volume fractions, is defined by BHE as

$$\beta = -\frac{2\ln V_{\rm f} + V_{\rm m}(3 - V_{\rm f})}{4V_{\rm m}^2} \tag{10}$$

and

$$K = \frac{1}{2}\left(1 + \sqrt{1 + \frac{4\rho^2 V_{\rm m}E_{\rm m}E_{\rm f}}{aE}\left(\frac{\zeta_{\rm d}}{\tau_{\rm s}^2}\right)}\right) \tag{11}$$

where $\zeta_{\rm d}$ is the debond toughness.

The upstream region (see Fig. 1) is so far away from the matrix crack tip such that the stress and strain fields are the same as those of the uncracked materials. The fiber and matrix axial stresses can be well approximated by

$$\sigma_{\rm f}^{\rm U}(z) = \frac{E_{\rm f}}{E}\sigma + \sigma_{\rm f}^{\rm I} \tag{12}$$

$$\sigma_{\rm m}^{\rm U}(z) = \frac{E_{\rm m}}{E}\sigma + \sigma_{\rm m}^{\rm I} \tag{13}$$

The energy relationship to evaluate the steady-state matrix cracking stress is expressed as (BHE [3])

$$\frac{1}{2}\int_{-\infty}^{\infty}\left[\frac{V_{\rm f}}{E_{\rm f}}\left(\sigma_{\rm f}^{\rm U} - \sigma_{\rm f}^{\rm D}\right)^2 + \frac{V_m}{E_{\rm m}}\left(\sigma_{\rm m}^{\rm U} - \sigma_{\rm m}^{\rm D}\right)^2\right]dz$$
$$+ \frac{1}{2\pi R^2 G_{\rm m}}\int_{-\infty}^{\infty}\int_{a}^{\overline{R}}\left(\frac{a\tau_{\rm i}^{\rm D}}{r}\right)^2 2\pi r\,dr\,dz = V_{\rm m}\zeta_{\rm m} \tag{14}$$

where $G_{\rm m}$ is the matrix shear modulus and $\zeta_{\rm m}$ is the matrix fracture toughness. Substituting the downstream stresses and the upstream stresses given, respectively, by Equations 12–13 and 3–4 with the debonded length $l_{\rm d}$ given by Equation 8, yields the standard algebraic form of cubic equation

$$B_3(\sigma + \sigma_m^{\rm I}E/E_{\rm m})^3 + B_1(\sigma + \sigma_m^{\rm I}E/E_{\rm m}) + B_0 = 0 \tag{15}$$

where

$$B_3 = \frac{a}{E_{\rm f}\tau_{\rm s}}\left(\frac{V_{\rm m}E_{\rm m}}{V_{\rm f}E}\right)^2 \tag{16-1}$$

$$B_1 = 12\frac{a\tau_{\rm s}}{\rho^2 E_{\rm f}} \tag{16-2}$$

$$B_0 = -8\frac{aV_{\rm f}E\tau_{\rm s}^2}{\rho^3 V_{\rm m}E_{\rm m}E_{\rm f}}\left[(K-1)^3+1\right] - 6V_{\rm m}\zeta_{\rm m} \tag{16-3}$$

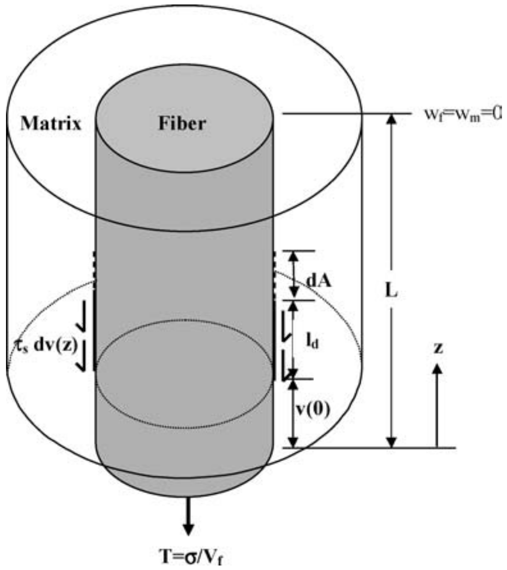

Figure 2 A composite-cylinder model.

The cubic equation of Equation 15 can be solved by Cardan's solution as

$$\sigma+\sigma_m^I E/E_m = \left\{-\frac{1}{2}\left(\frac{B_0}{B_3}\right)+\left[\frac{1}{4}\left(\frac{B_0}{B_3}\right)^2+\frac{1}{27}\left(\frac{B_1}{B_3}\right)^3\right]^{1/2}\right\}^{1/3} - \left\{\frac{1}{2}\left(\frac{B_0}{B_3}\right)+\left[\frac{1}{4}\left(\frac{B_0}{B_3}\right)^2+\frac{1}{27}\left(\frac{B_1}{B_3}\right)^3\right]^{1/2}\right\}^{1/3} \quad (17)$$

The well-known ACK stress σ_1 for larger slipping fibers can be recovered from Equation 17 by setting $\zeta_d = 0$ and dropping the $(1/\rho)$ terms due to the negligence of matrix shear deformation above the slipping region:

$$\sigma_1 = \left(\frac{6V_f^2 E_f E^2 \tau \zeta_m}{a V_m E_m^2}\right) \quad (18)$$

The stress σ_0 for no-debond interface given by Aveston and Kelly (AK) is given by

$$\sigma_0 = \left(\frac{\rho V_f E_f E}{a E_m}\zeta_m\right)^{1/2} \quad (19)$$

From definitions of the ACK stress σ_1 by Equation 18 and the AK stress σ_0 by Equations 19, Equation 17 can be arranged into a dimensionless form as

$$\frac{\sigma_{mc}+\sigma_m^I E/E_m}{\sigma_0} = \{X+[X^2+(Y^{18}/729)]^{1/2}\}^{1/3} - \{-X+[X^2+(Y^{18}/729)]^{1/2}\}^{1/3} \quad (20)$$

where

$$Y = \frac{\sigma_1}{\sigma_0} = \left(\frac{36 a V_f E}{\rho^3 V_m^2 E_m E_f \zeta_m}\right)^{1/6} \tau_s^{1/3} \quad (21)$$

and

$$X = \frac{Y^9}{54}+\frac{Y^3}{2}+\frac{1}{54} \times \left\{\left[\frac{Y^6}{4}+36\left(\frac{V_f}{\rho V_m}\right)\left(\frac{\zeta_d}{\zeta_m}\right)\right]^{1/2} - \frac{Y^3}{2}\right\}^3 \quad (22)$$

For purely frictional interface (i.e. $\zeta_d = 0$), Equation 20 reduces to the BHE result as

$$\frac{\sigma_{mc}+\sigma_m^I E/E_m}{\sigma_0} = \left\{\frac{Y^9}{54}+\frac{Y^3}{2}+\left[\left(\frac{Y^9}{54}+\frac{Y^3}{2}\right)^2+\frac{Y^{18}}{729}\right]^{1/2}\right\}^{1/3} - \left\{-\frac{Y^9}{54}-\frac{Y^3}{2}+\left[\left(\frac{Y^9}{54}+\frac{Y^3}{2}\right)^2+\frac{Y^{18}}{729}\right]^{1/2}\right\}^{1/3} \quad (23)$$

By substituting Equation 20 of the dimensionless form of critical matrix cracking stress into Equation 8, the debonded length l_d can be expressed as a dimensionless form of

$$\frac{l_d}{a} = \frac{3}{\rho Y^3}\left(\frac{\sigma+\sigma_m^I E/E_m}{\sigma_0}\right) - \frac{1}{2\rho}\left[\left(1+\frac{144}{Y^6}\left(\frac{V_f}{\rho V_m}\right)\left(\frac{\zeta_d}{\zeta_m}\right)\right)^{1/2}+1\right] \quad (24)$$

Sutcu and Hillig [8] have used an energy balance approach to treat the fiber/matrix debonding problem and developed a critical matrix cracking stress formulation that is identical to Equation 20 except for the expression of X by

$$X = \frac{Y^9}{54}+\frac{Y^3}{2}+4\left(\frac{V_f \zeta_d}{\rho V_m \zeta_m}\right)^{3/2} \quad (25)$$

TABLE I Properties of SiC/LAS composite

	SiC/LAS
E_f	200 Gpa
E_m	85 Gpa
ν_m	0.25
a	8 μm
ζ_m	47 J/m^2
τ_s	1–2 MPa

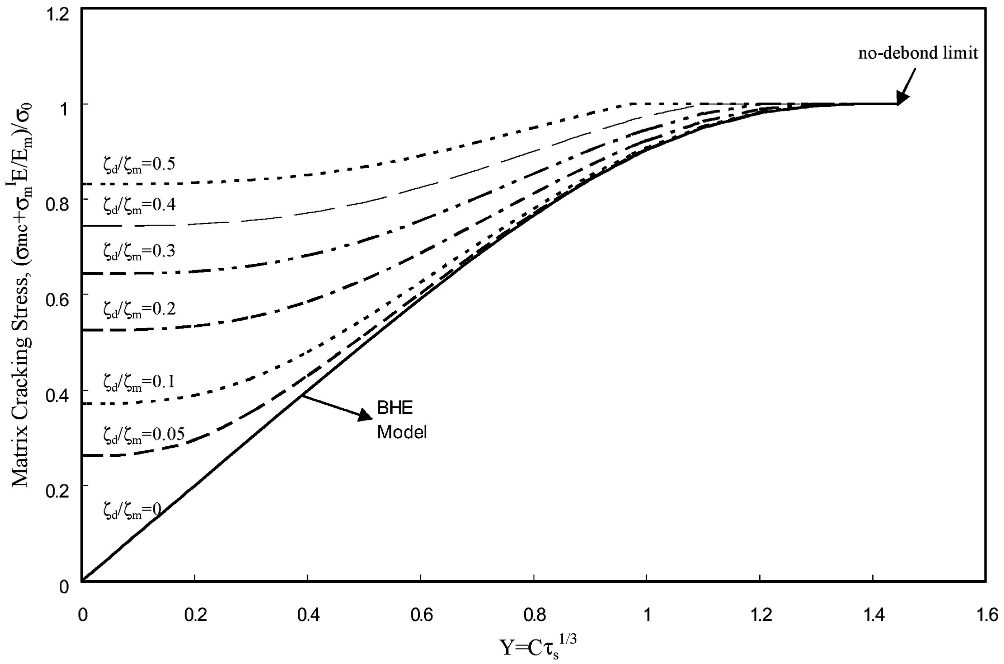

Figure 3 $[\sigma_{mc} + (E/E_m)\sigma_m^I]/\sigma_0$ vs. $Y = C\tau_s{}^{1/3}$ at different ζ_d/ζ_m for SiC/LAS at $V_f = 0.5$.

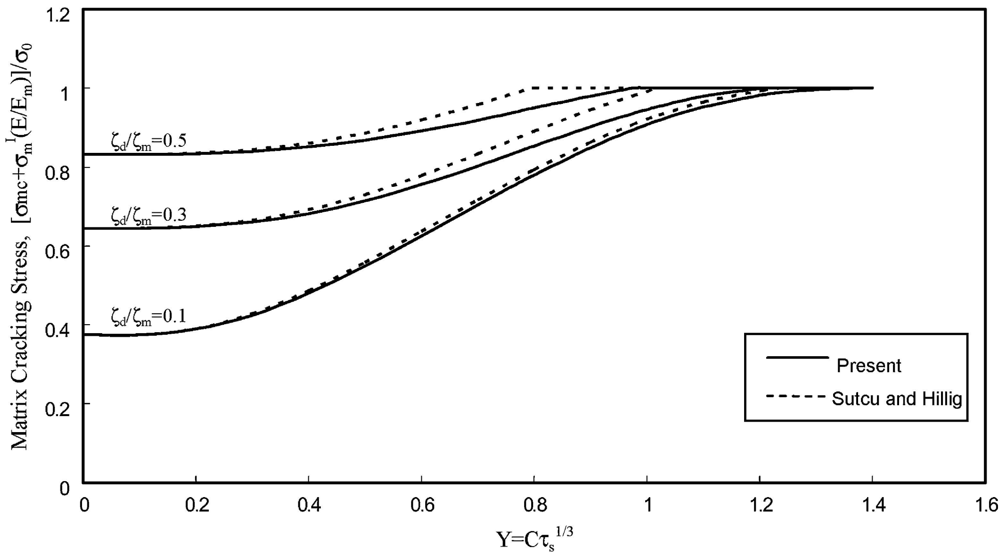

Figure 4 Comparison of the results between the present analysis and Sutcu and Hillig for $[\sigma_{mc} + (E/E_m)\sigma_m^I]/\sigma_0$ vs. $Y = C\tau_s{}^{1/3}$ at different ζ_d/ζ_m for SiC/LAS at $V_f = 0.5$.

The comparison of the relative critical matrix cracking stress, $[\sigma_{mc} + (E/E_m)\sigma_m^I]/\sigma_0$, as a function of the dimensionless friction parameter Y=C$\tau_s{}^{1/3}$ for different relative debond toughness, ζ_d/ζ_m, for SiC/LAS composite (see Table I) is illustrated in Fig. 3. As similar to the case of the purely frictional interface (i.e. $\zeta_d = 0$), that is equivalent to the BHE result, the present analysis provides the smooth link between the result of frictionless bonded interface by Stang and Shan and the no-debond result by AK. The comparison between the present analysis and Sutcu and Hillig is illustrated in Fig. 4, in which the present analysis provides the tangential approaches to the no-debond limits. On the other hand, the curves predicted by Sutcu and Hillig for bonded interfaces are secant lines at the no-debond limits, as shown in Fig. 4. Compared to the Sutcu and Hillig model, the results by present analysis are shown to be more rational on the perspective of mathematical rigorousness.

A fracture mechanics approach in which the crack-wake debonding process is treated as a particular crack propagation problem is adopted in the present analysis. The newly derived closed-form solution of critical matrix cracking stress given by Equation 20 represents more general interfacial properties of composite. Similar to the BHE model that provides results to bridge between the large-slip result by ACK and the no-slip result by AK, the present analysis provides the smooth link between the result of frictionless bonded interface by Stang and Shan and the no-debond result by AK. The differences of mathematical modeling and theoretical results between the present analysis and Sutcu and Hillig have been shown and discussed.

References

1. W. A. CURTIN, *Comprehensive Composite Materials, Elsevier Science*, **4** (2000) 47.
2. V. KOSTOPOULOS and Y. Z. PAPPAS, *Comprehensive Composite Materials, Elsevier Science Ltd.* **4** (2000) 95.
3. B. BUDIANSKY, J. W. HUTCHINSON and A. G. EVANS, *J. Mech. Phys. Solids.* **34**(2) (1986) 167.
4. Y. C. CHIANG, *Engi. Frac. Mech.* **65**(1), (2000) 15.
5. J. AVESTON, G. A. COOPER and A. KELLY, in Properties of Fiber Composites: Conference on Proceedings (National Physical Laboratory, IPC, England, 1971) p. 15.
6. J. AVESTON and A. KELLY, *J. Mat. Sci.* **8**(3) (1973) 352.
7. H. STANG and S. P. SHAH, *ibid.* **21**(3) (1986) 953.
8. M. SUTCU and W. B. HILLIG, *Acta Metall.* **38**(12) (1990) 2653.
9. Y. C. GAO, Y. W. MAI and B. COTTERELL, *J., Appl. Math.Phys. (ZAMP)* **39**(7), 550–572 (1988).
10. Y. C. CHIANG, *J Comp. Sci. & Techn.*, **61**(12) (2001) 1743.

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